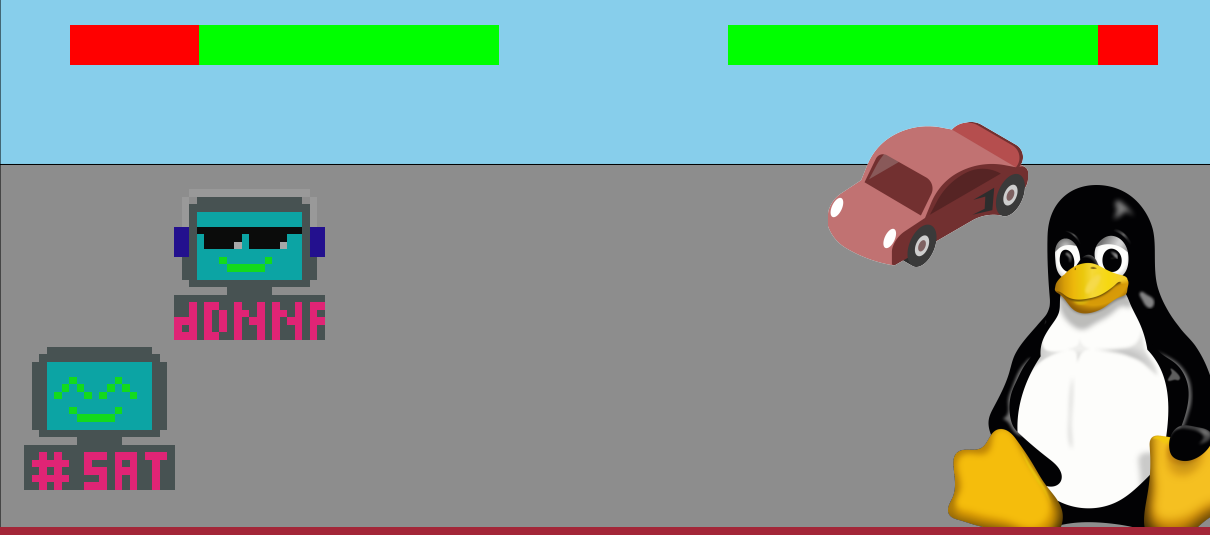

Tackling Expressive FM Constructs with Pseudo-Boolean d-DNNF Compilation

FOSD'26 | Chico Sundermann, Stefan Vill, Elias Kuiter, **Sebastian Krieter**, Thomas Thüm, Matthias Tichy | 24.03.2026

universität ulm

Technische Universität Braunschweig

Introduction Product Lines

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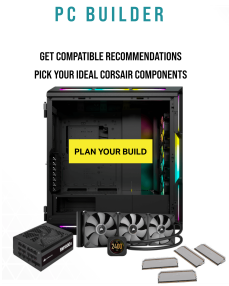

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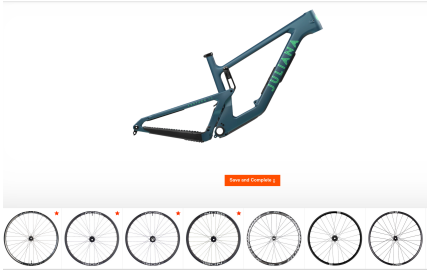

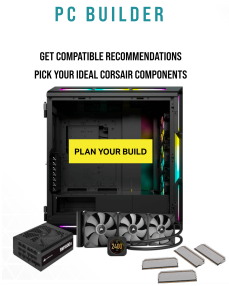

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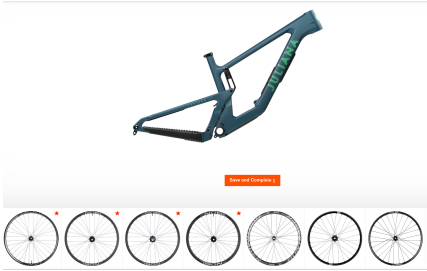

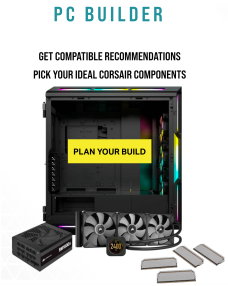

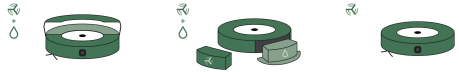

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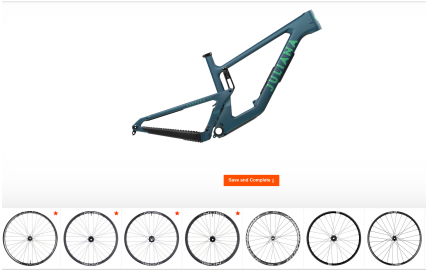
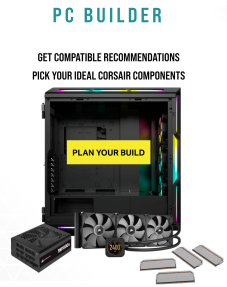
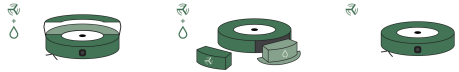
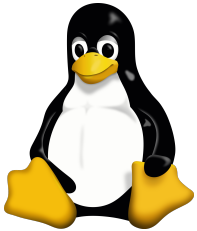
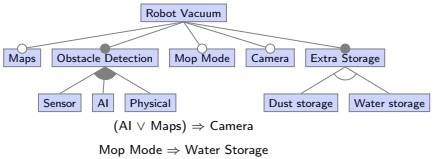


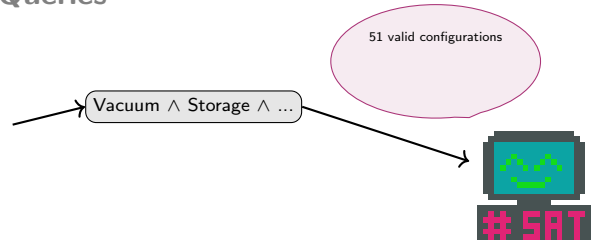




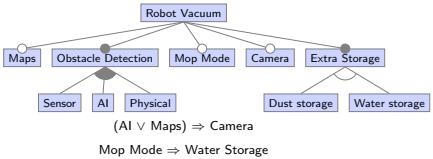


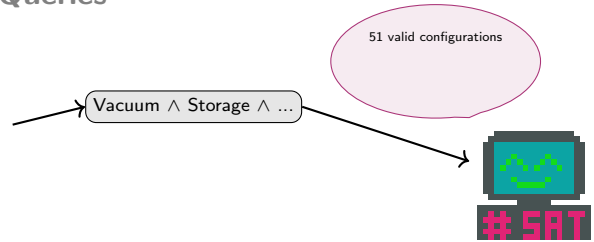
Scalability Issues Multitude of Queries





Scalability Issues Multitude of Queries

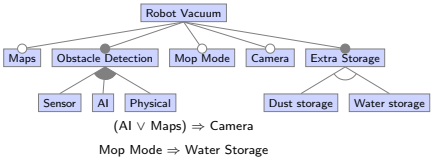


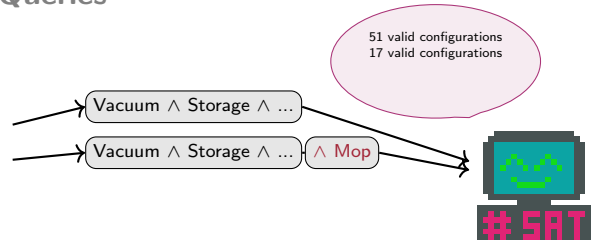


EMSE'23

Most systems: Few seconds of runtime

Scalability Issues Multitude of Queries





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Scalability Issues

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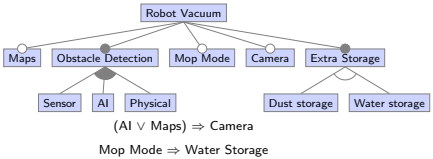
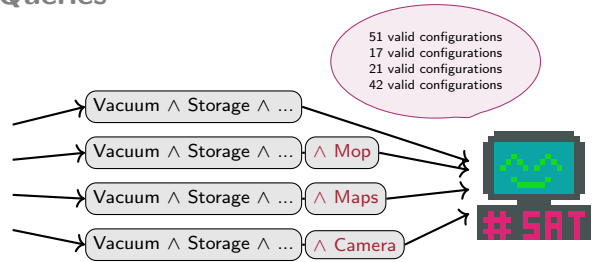




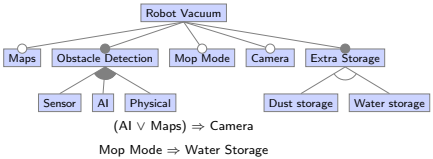
EMSE'23

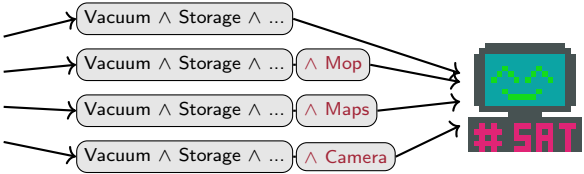
Most systems: Few seconds of runtime

Type	Number of Queries	Repeatability
Features	$\mathcal{O}(\lvert F\rvert)$	Static
	$\mathcal{O}(\lvert F\rvert)$	Static
	$\mathcal{O}(\lvert F\rvert)$	Static
	$\mathcal{O}(\lvert F\rvert)$	Static
	$\mathcal{O}(\lvert F\rvert)$	Static
	$\mathcal{O}(\lvert F\rvert)$	Static
	$\mathcal{O}(\lvert F\rvert)$	Static
	$\mathcal{O}(\lvert F\rvert)$	Static
Partial Configurations	$\mathcal{O}(\lvert F\rvert^2)$	Dynamic
	$\mathcal{O}(\lvert F^2\rvert)$	**Dynamic**
	$\mathcal{O}(\lvert F\rvert)$	**Dynamic**
	$\mathcal{O}(\lvert F^2\rvert)$	**Dynamic**
Formula	1	**Dynamic**
	1	**Dynamic**

AMAI'24

Scalability Issues Multitude of Queries





EMSE'23

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	$\mathcal{O}(\lvert F\rvert)$	Static
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	$\mathcal{O}(\lvert F\rvert)$	Static
	$\mathcal{O}(\lvert F\rvert)$	Static
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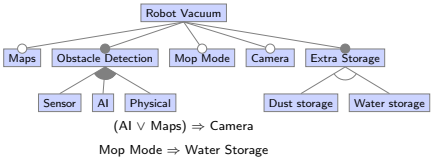
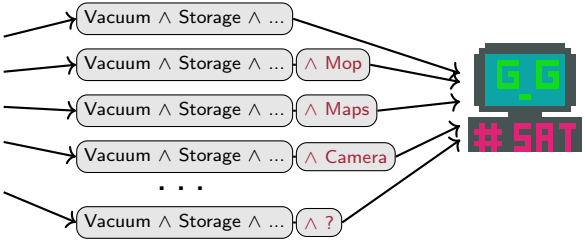
AMAI'24

Scalability Issues Multitude of Queries





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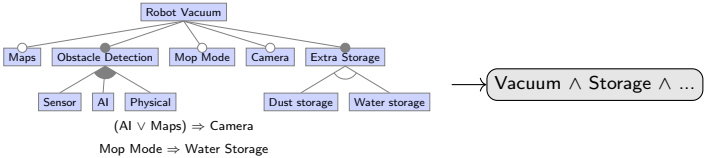
Most systems: Few seconds of runtime

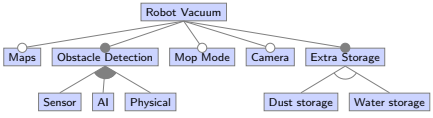
EMSE'23

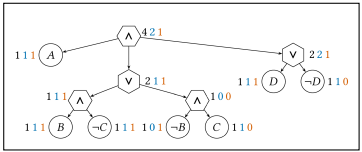
AMAI'24

Knowledge Compilation d-DNNF

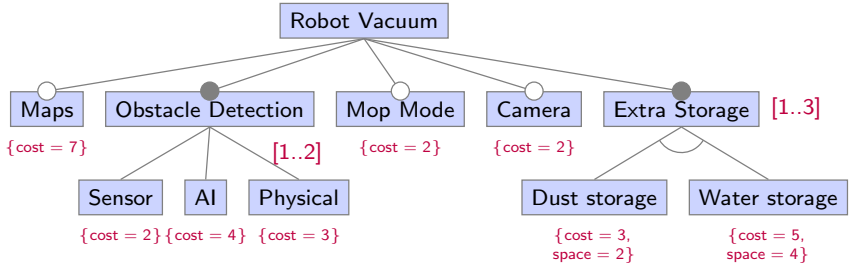


Knowledge Compilation d-DNNF





Coping with Expressive Constructs



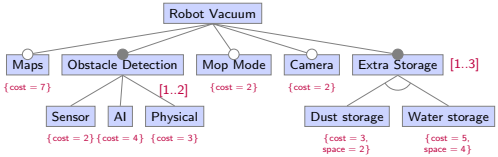
$$(\text{AI} \vee \text{Maps}) \Rightarrow \text{Camera}$$

$$\text{Mop Mode} \Rightarrow \text{Water Storage}$$

$$\textbf{sum}(\text{cost}) < 15$$

$$\text{"Dust storage".space} + \text{"Water storage".space} \leq 6$$

Coping with Expressive Constructs



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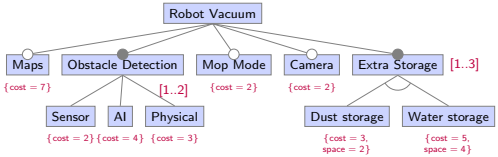
Coping with Expressive Constructs

Pseudo-Boolean d-DNNF Compilation



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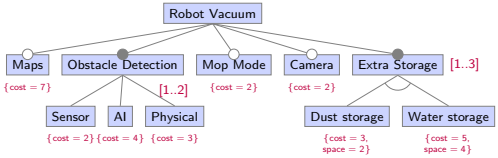
Pseudo-Boolean translation →

$$\mathsf{Sensor} + \mathsf{AI} + \mathsf{Physical} \geq 1$$
$$\mathsf{Sensor} + \mathsf{AI} + \mathsf{Physical} \leq 2$$
$$\sum \mathsf{cost} \cdot \mathsf{feature} \leq 15$$

Coping with Expressive Constructs

Pseudo-Boolean d-DNNF Compilation



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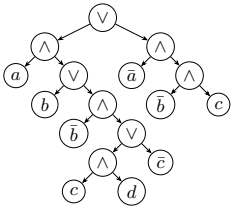
Pseudo-Boolean d-DNNF compilation



Pseudo-Boolean Encoding Excerpt

Construct	Boolean	Pseudo-Boolean
$opt^p_{c_1..c_n}$	$\bigwedge_{i=1}^{n}(c_i \Rightarrow p)$	$n \cdot p + \sum_{i=1}^{n} -c_i \geq 0$
$mand^p_{c_1..c_n}$	$\bigwedge_{i=1}^{n}(c_i \Leftrightarrow p)$	$n \cdot p + \sum_{i=1}^{n} -c_i = 0$
$or^p_{c_1..c_n}$	$p \Leftrightarrow (\bigvee_{i=1}^{n} c_i)$	$n \cdot p + \sum_{i=1}^{n} -c_i \geq 0$
		$-p + \sum_{i=1}^{n} c_i \geq 0$
$alt^p_{c_1..c_n}$	$p \Leftrightarrow (\bigvee_{i=1}^{n} c_i \wedge \bigwedge_{i<j}(\neg c_i \vee \neg c_j))$	$p + \sum_{i=1}^{n} -c_i = 0$

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$opt^p_{c_1..c_n}$	$\bigwedge_{i=1}^n (c_i \Rightarrow p)$	$n \cdot p + \sum_{i=1}^n -c_i \geq 0$
$mand^p_{c_1..c_n}$	$\bigwedge_{i=1}^n (c_i \Leftrightarrow p)$	$n \cdot p + \sum_{i=1}^n -c_i = 0$
$or^p_{c_1..c_n}$	$p \Leftrightarrow (\bigvee_{i=1}^n c_i)$	$n \cdot p + \sum_{i=1}^n -c_i \geq 0$ $-p + \sum_{i=1}^n c_i \geq 0$
$alt^p_{c_1..c_n}$	$p \Leftrightarrow (\bigvee_{i=1}^n c_i \wedge \bigwedge_{i<j} (\neg c_i \vee \neg c_j))$	$p + \sum_{i=1}^n -c_i = 0$
$[a..b]^p_{c_1..c_n}$	$\text{enum}([a..b]^p_{c_1..c_n})$	$n \cdot p + \sum_{i=1}^n -c_i \geq 0$ $-a \cdot p + \sum_{i=1}^n c_i \geq 0$ $\sum_{i=1}^n -c_i \geq -b$
$f[a..b]$	$\text{enum}([a..b]^{cr}_{c_1,..,c_b})$	$pb([a..b]^{cr}_{c_1,..,c_b})$
$\phi(A) \star \psi(A')$	$\text{enum}(\phi(A) \star \psi(A'))$	$pb(\phi(A)) \star pb(\psi(A'))$
$\rightarrow\ c_i$		c_i
$\rightarrow\ a_i = (v_i, f_i)$		$v_i \cdot f_i$
$\rightarrow\ (c_1 + \sum_{i=1}^n k_i \cdot x_i) +$ $(c_2 + \sum_{i=1}^m l_i \cdot y_i)$		$(c_1 + c_2) + \sum_{i=1}^n k_i \cdot x_i$ $+ \sum_{i=1}^m l_i \cdot y_i$
$\rightarrow\ \ldots$		$\ldots$

Pseudo-Boolean d-DNNF Compilation — Concept

Idea

Reuse trace from exhaustive DPLL on Pseudo-Boolean formula

Pseudo-Boolean d-DNNF Compilation — Concept

Idea

Reuse trace from exhaustive DPLL on Pseudo-Boolean formula

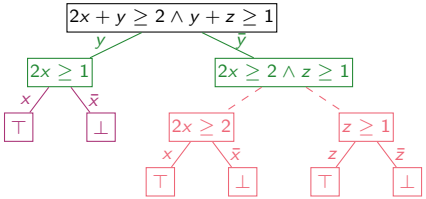

Figure: Trace for Exhaustive Pseudo-Boolean DPLL

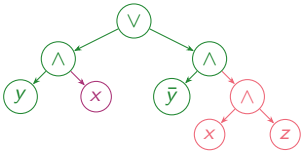

Figure: d-DNNF Corresponding to Traversal in Trace

Evaluation Setup

What is the performance of pseudo-Boolean d-DNNFs?

Dataset	#Models	#Features
Literature	76	100–80,258
Industrial	2	612–645
Isolated Concepts	2,577	1–8,000
Synthesized Models	2,700	80–2,767

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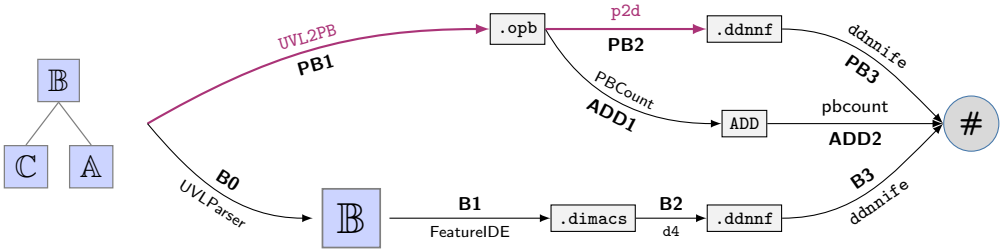
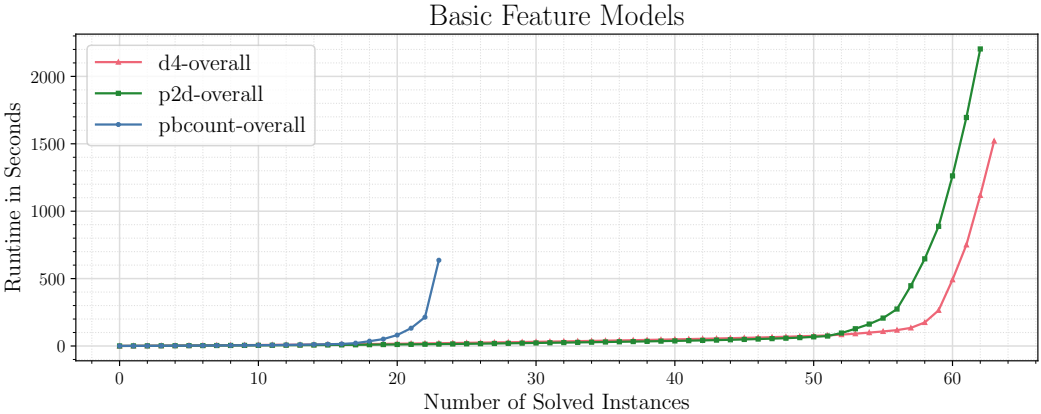
Pipelines:

- **p2d**: Pseudo-Boolean with `p2d`
- **pbcount**: Pseudo-Boolean with `pbcount`
- **d4**: Boolean with `d4`



Evaluation Performance vs State of the Art



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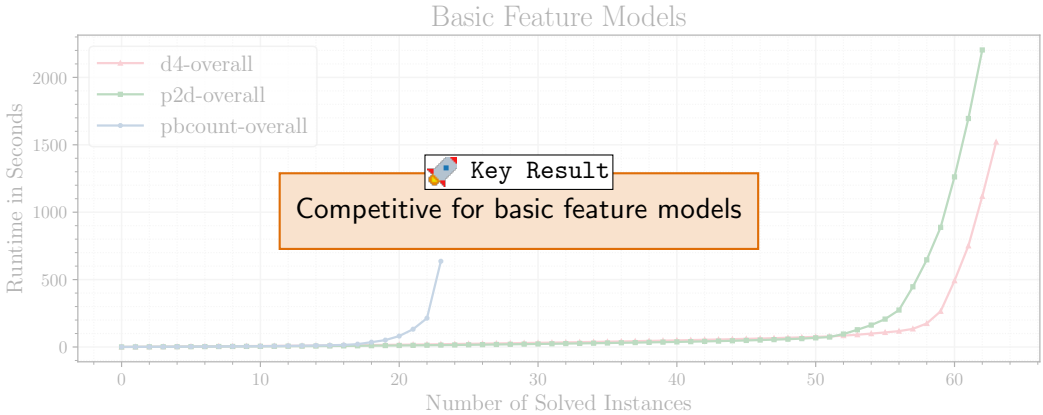
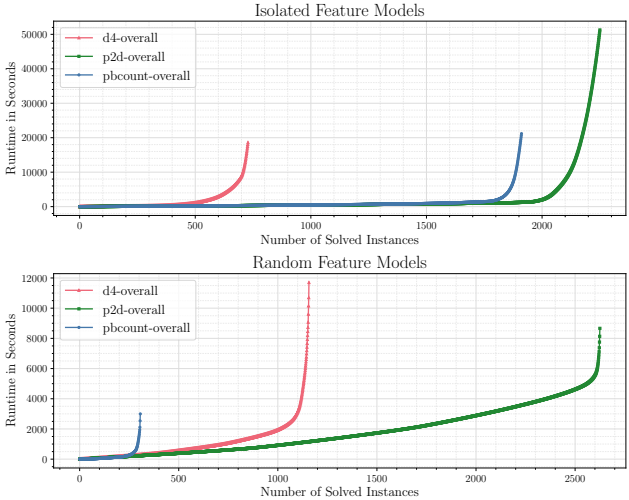


Evaluation

Performance vs State of the Art



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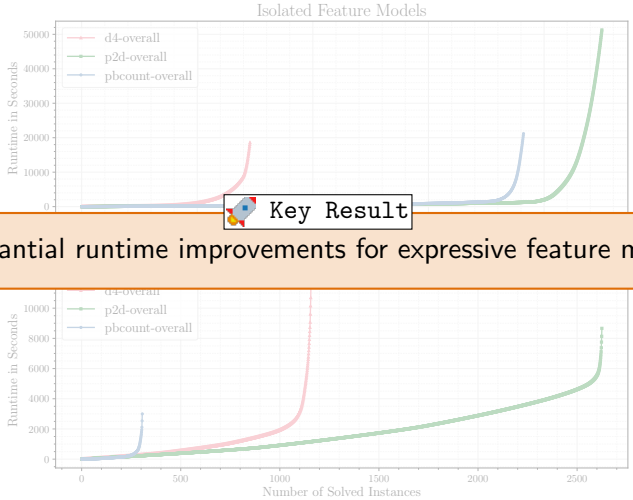
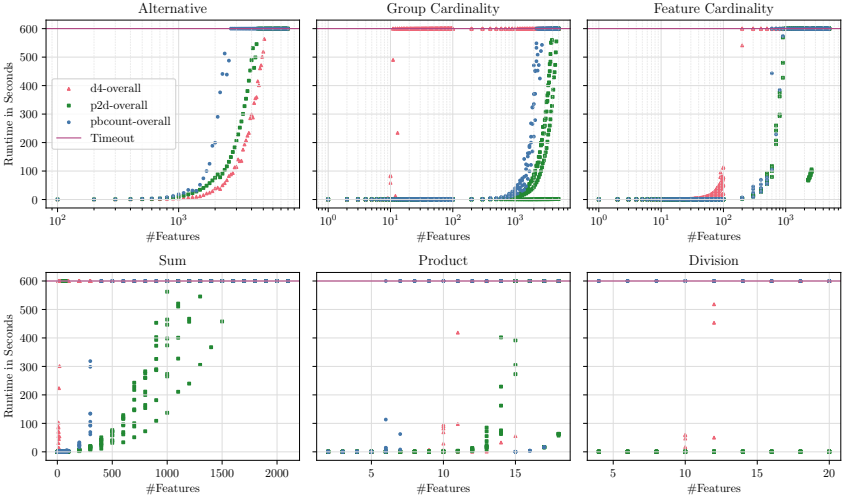
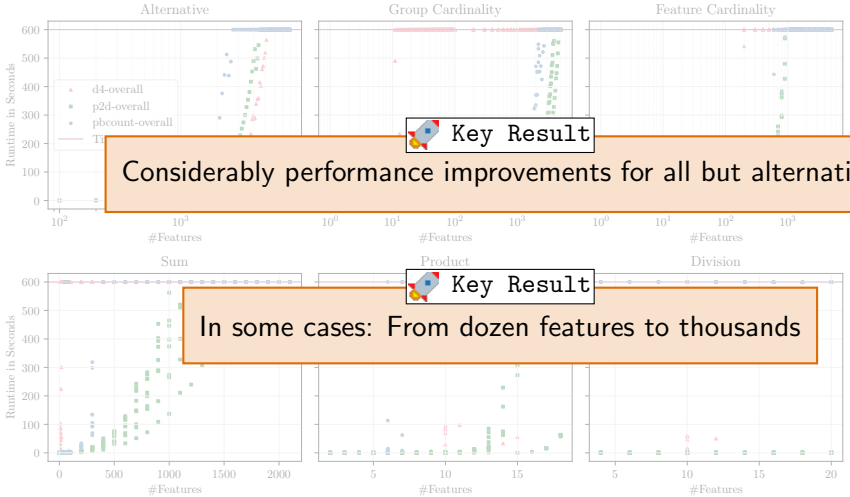


Evaluation

Performance: Isolated Concepts

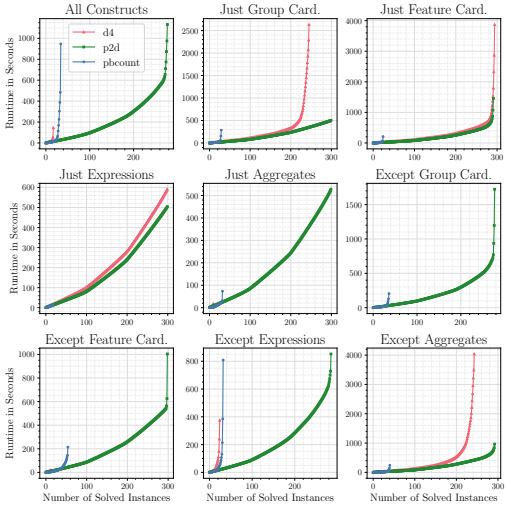


Evaluation — Performance: Isolated Concepts



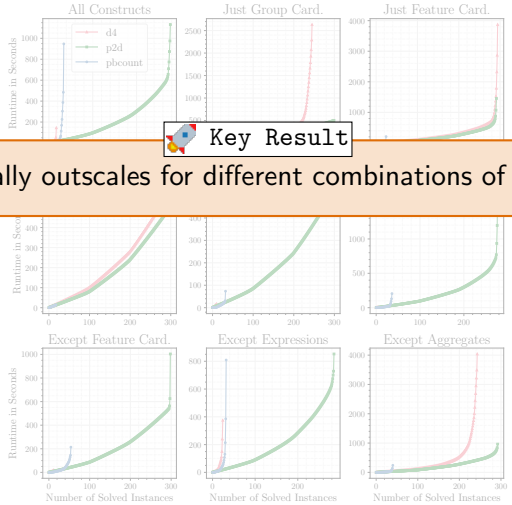
Evaluation

Performance: Combinations of Concepts



Evaluation

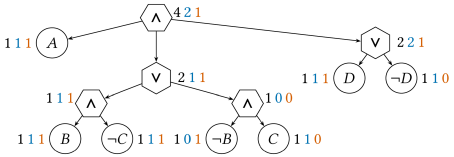
Performance: Combinations of Concepts



Key Result

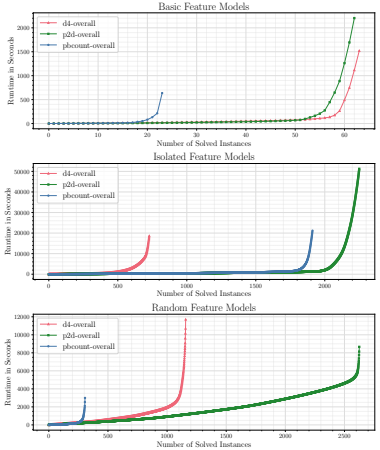
Substantially outscales for different combinations of constructs

Conclusion

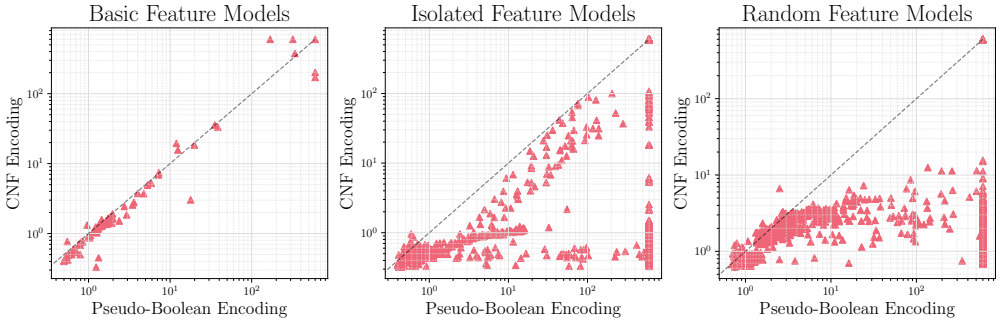


Compilation:
`p2d`

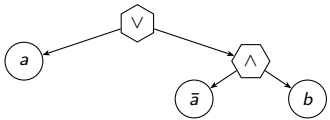
Reasoning:
`ddnnife`



Evaluation Encoding Efficiency



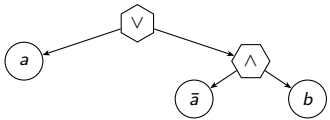
d-DNNFs Behind the Scenes



deterministic
decomposable
negation
normal
form

[2] Darwiche, A Compiler for Deterministic, Decomposable Negation Normal Form, AAAI'02

d-DNNFs Behind the Scenes



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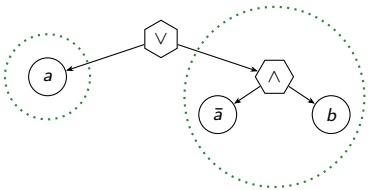

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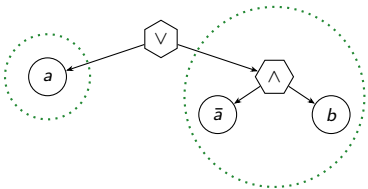

deterministic (Sum for $\vee$)
decomposable
negation
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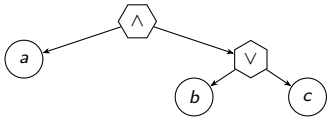
d-DNNFs Behind the Scenes



deterministic (Sum for $\vee$)
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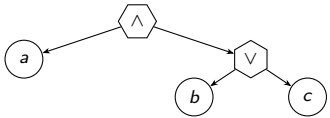
d-DNNFs Behind the Scenes



$\{a\}$

1

$\{b, c\}$
$\{\bar{b}, c\}$
$\{b, \bar{c}\}$

3

deterministic (Sum for $\vee$)
decomposable
negation
normal
form

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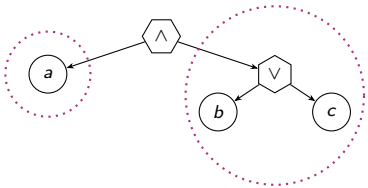
d-DNNFs Behind the Scenes



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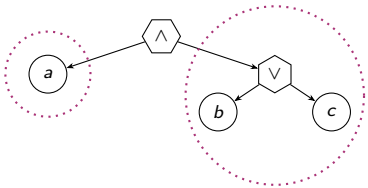
d-DNNFs Behind the Scenes



deterministic (Sum for $\vee$)
decomposable (Product for $\wedge$)
negation
normal
form

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