

Sampling-based Testing for Partial Quantum State Tomography

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Qubits and Qubit States

- Quantum computations are based on **operations on qubits**.
- Qubits may have different **physical** realizations in a quantum computer.
- **Mathematically**, the **state** of a qubit ψ can be defined as a two-dimensional unit vector in the complex vector space (Hilbert-Space):

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \text{ where } \alpha, \beta \in \mathbb{C} \text{ such that } |\alpha|^2 + |\beta|^2 = 1 \quad (\text{Dirac Notation})$$

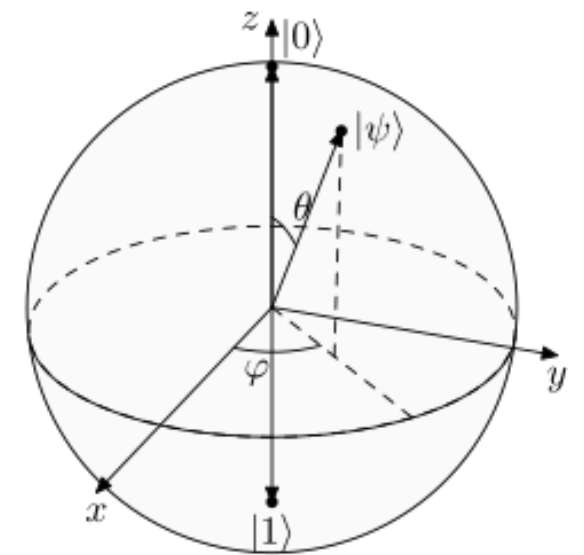
Measuring Qubit States

- A single qubit can (theoretically) encode an **infinite amount of information**.
- But: The exact state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ of a qubit ψ (i.e., the values of α and β) is **not directly accessible by any readout operation**.
- Instead: Measurement on $|\psi\rangle$ yields classical bit '0' with **probability** $p_0 = |\alpha|^2$ and classical bit '1' with **probability** $p_1 = |\beta|^2$.
- After readout, the quantum state of qubit $|\psi\rangle$ **collapses** into the classical bit state corresponding to the measurement result.

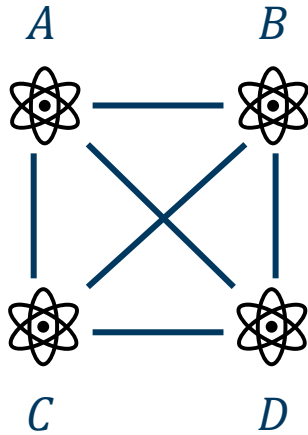
Quantum State Tomography

- Quantum state tomography algorithms define **series of projective measurement settings** to extract (partial) information from qubit states.
- Example: Pauli operators X, Y, Z utilize geometrical interpretation of qubit states to measure their three characterizing directions:

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right) |0\rangle + e^{i\varphi} \sin\left(\frac{\theta}{2}\right) |1\rangle \quad (\text{Bloch Sphere})$$



Full Tomography of Multi-Qubit States

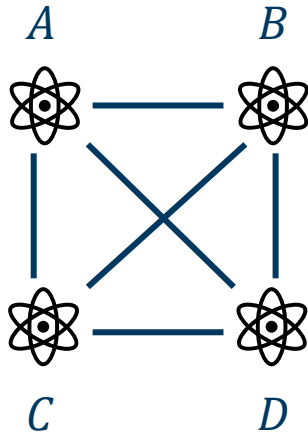


	A	B	C	D
1.	X_A	X_B	X_C	X_D
2.	X_A	X_B	X_C	Y_D
...
81.	Z_A	Z_B	Z_C	Z_D

Full tomography scheme of 4-qubit states requires $3^4 = 81$ settings

- Performing state tomography for an **N -qubit quantum system** on every qubit individually is not feasible.
- Reason: **Entanglement** (i.e., correlations between measurement result probabilities of different qubits).

Partial State Tomography for Large-Scale Quantum Systems

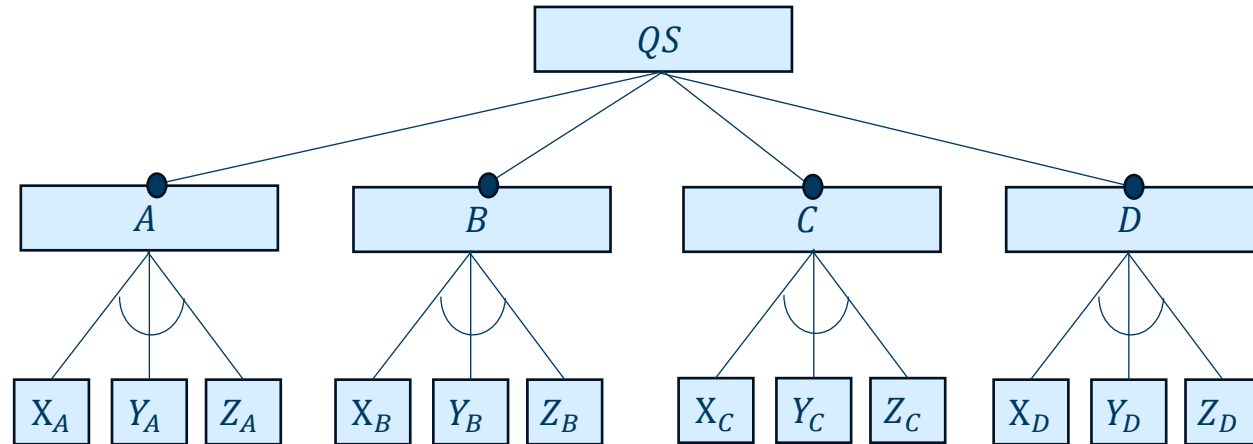
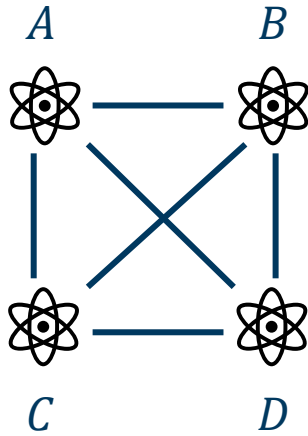


	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1.	X_A	X_B	X_C	X_D
2.	Z_A	Y_B	Y_C	X_D
3.	Y_A	Z_B	Z_C	X_D
4.	Y_A	Y_B	X_C	Y_D
5.	X_A	Z_B	Y_C	Y_D
6.	Z_A	X_B	Z_C	Y_D
7.	Z_A	Z_B	X_C	Z_D
8.	Y_A	X_B	Y_C	Z_D
9.	X_A	Y_B	Z_C	Z_D

Partial tomography scheme of 2-body marginal 4-qubit states requires 9 settings

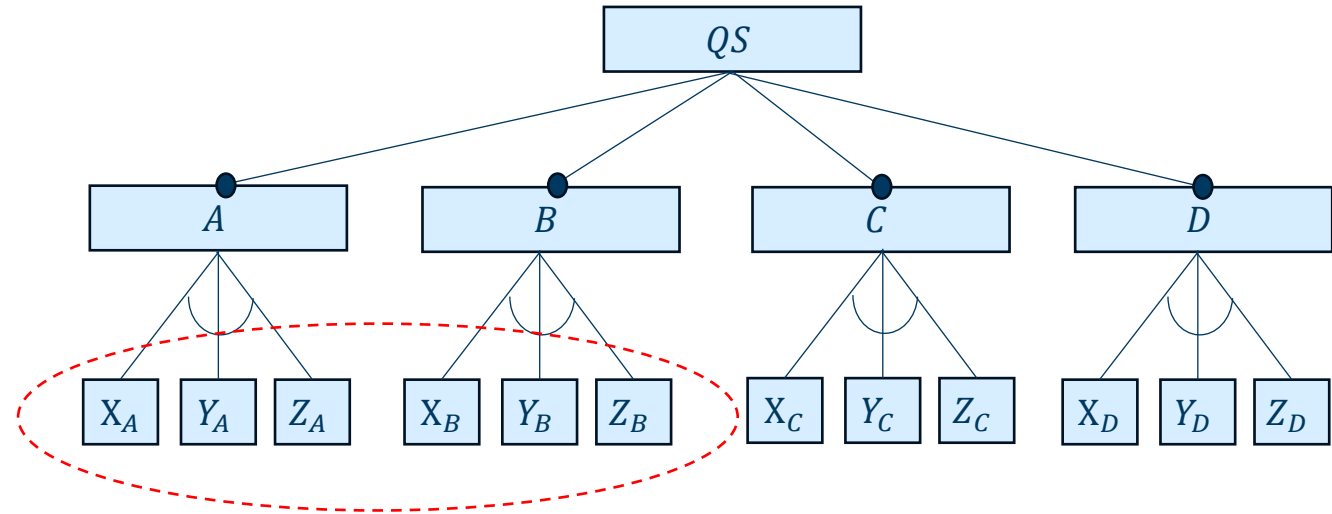
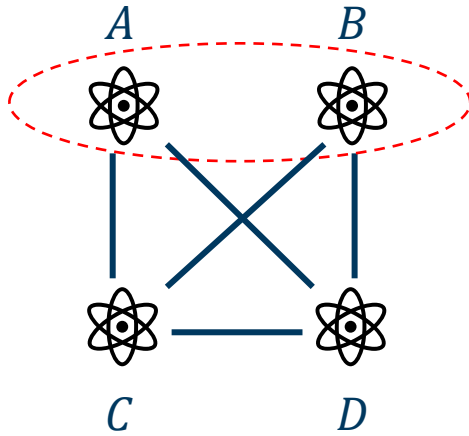
- Tomography of 2-body marginal states reducible to minimum edge-clique covering problem.

Sampling-based Testing for Partial Quantum State Tomography



- Tomography scheme generation of k -body marginal states for any N based on feature-model encoding and k -wise sampling.
- But: This only works for fully-meshed topologies without constraints.

Ongoing Research



- Pairwise coverage of **at least one** instead of **all possible** combinations of settings in case of independent qubits => Talk by Mathis Weiß.
- Fermionic systems impose additional constraints on valid combinations of settings => Submitted to QSW.
- ...

Thank You!

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